Name ______________________________

Honors Pre-Calculus FINAL EXAM REVIEW

All of the topics below are covered on the final exam to some extent. Please use this as a guide to help you study your notes, homework, and past quizzes and tests. I will provide you with your folder of tests soon.

Please refer to your midterm exam study guide for additional notes on the topics we studied during the first semester:

Domain and Range of Functions

Composite Functions

Inverse Functions

Linear Functions

Matrices (dimensions, addition/subtraction, scalar multiplication, multiplication, solving systems of 2 and 3 variable equations with matrices)

Parent Functions

Piecewise Functions

Transformations

Analyzing graphs (odd/even, maximum/minimum, increasing/decreasing/constant, continuous/discontinuous, end behavior, zeros/roots/x-intercepts, y-intercept, f(x) notation, vertical line test)

Solving Quadratics

Polynomial Functions (degree, leading coefficient, end behavior, roots and multiplicity, rational root theorem, Decartes Rule of Signs)

Rational Functions (domain, asymptotes – vertical, horizontal, oblique/slant, holes)

Radical Functions

Fractional Exponents

Exponential Functions

Solving Exponential Equations

Logarithmic Functions, log properties, solving log equations

Topics that we studied during the second semester:

Circles

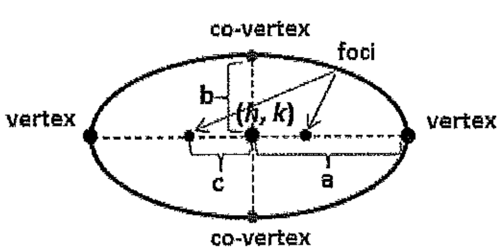
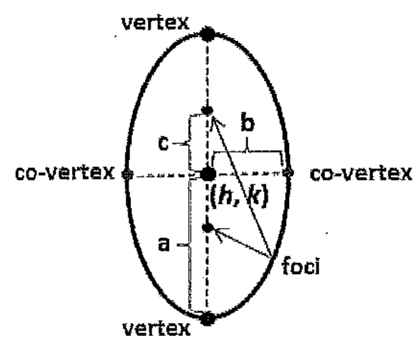
Center (h,k)   Any point on the circle (x,y)   r=radius

$$\text{Standard form: } (x-h)^2 + (y-k)^2 = r^2$$

$$\text{General form: } x^2 + y^2 + Dx + Ey + F = 0$$

- Be able to:
  - Identify the center and radius.
  - Write the equation for a circle in standard form given its center and radius.
  - Convert an equation from standard form to general form.
  - Convert an equation from general form to standard form.
  - Write the equation of a circle given three points (plugging in, using matrices, etc.)

Ellipses

<table>
<tr><th>Horizontal Ellipse</th><th>Vertical Ellipse</th></tr>
<tr><td>At (0, 0): $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$</td><td>At (0, 0): $\frac{x^2}{b^2}+\frac{y^2}{a^2}=1$</td></tr>
<tr><td>General: $\frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}=1$
$a^2-b^2=c^2$</td><td>General: $\frac{(x-h)^2}{b^2}+\frac{(y-k)^2}{a^2}=1$
$a^2-b^2=c^2$</td></tr>
<tr><td>Center: (h,k) Foci: $(h\pm c,k)$
Vertices: $(h\pm a,k)$ Co-Vertices: $(h,k\pm b)$</td><td>Center: (h,k) Foci: $(h,k\pm c)$
Vertices: $(h,k\pm a)$ Co-Vertices: $(h\pm b,k)$</td></tr>
<tr><td>
</td><td>
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- Be able to:
  - Identify the center of the ellipse.
  - Determine the length of the major and minor axes from the equation.
  - Figure out all of the variables (a^2,a, b^2, b, c, h, k) and plug them in to the forms above to identify the vertices, co-vertices, foci, etc.
  - Graph an ellipse.
  - Write the equation of an ellipse given different information.

Practice:

1. Write an equation of a circle with a radius of 8 and a center of (4, 3).

$(x-4)^2+(y-3)^2=64$

2. Write an equation of a circle with a radius of 6 and a center of (-3, -8).

$(x+3)^2+(y+8)^2=36$

3. Write the equation of a circle given the center is (2, -5) with a radius of 7.

$(x-2)^2+(y+5)^2=49$

4. Find the center and radius. $(x-1)^2+(y+4)^2=81$

$(1,-4)\quad r=9$

5. Find the radius and center of: $(x-5)^2+(y-2)^2=20$

$(5,2)\quad r=2\sqrt{5}$

6. Convert to general form. $(x-2)^2+(y+3)^2=9$

$x^2-4x+4+y^2+6y+9=9$

$x^2+y^2-4x+6y+4=0$

7. Convert to general form. $(x+4)^2+(y-2)^2=1$

$x^2+8x+16+y^2-4y+4=1$

$x^2+y^2+8x-4y+19=0$

8. Convert to standard form, then find the center and radius. $x^2-8x+y^2+11=0$

$x^2-8x+16+y^2=-11+16$

$(x-4)^2+y^2=5\quad$ ctr: $(4,0)\quad r=\sqrt{5}$

9. Convert to standard form, then find the center and radius. $x^2+y^2+4x-6y=-4$

$x^2+4x+\underline{4}+y^2-6y+\underline{9}=-4+\underline{4}+\underline{9}$

$(x+2)^2+(y-3)^2=9\quad$ ctr $(-2,3)\quad r=3$

10. Convert to standard form, then find the center and radius. $3x^2=9-3y^2-6y$

$\dfrac{3x^2+3y^2+6y-9=0}{3}$

$x^2+y^2+2y=3$

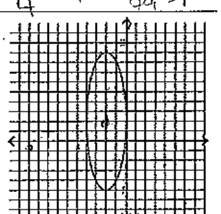
$x^2+y^2+2y+\underline{1}=3+\underline{1}$

$x^2+(y+1)^2=4$

ctr $(0,-1)\quad r=2$

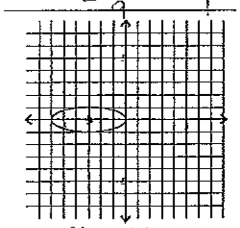
Write the standard form of the equation of the ellipses shown below.

V 5. $\dfrac{(x+2)^2}{4}+\dfrac{(y-2)^2}{49}=1$



ctr: $(-2,2)$

H 6. $\dfrac{(x-3)^2}{9}+\dfrac{y^2}{1}=1$



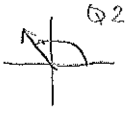
ctr $(3,0)$

Angles in the Coordinate Plane

- Positive/Negative angle measures
- Understand quadrants and ASTC
- Standard Position

Ex. Sketch a graph of the following angles and state the quadrant they are in:

A. 119°

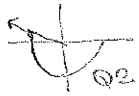


Q2

B. 288°

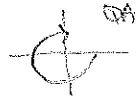
Q4

C. −190°



Q2

D. −270°



QA

- Coterminal Angles

Ex. State three co-terminal angles for A. 15°

375°
735°
−345°

B. $\frac{7\pi}{6}$ $\quad \frac{19\pi}{6},\ \frac{31\pi}{6},\ -\frac{5\pi}{6}$

- Rotations

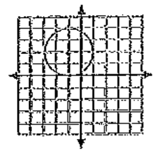
½ rotation = 180°

1 rotation = 360°

2 rotations = 720°

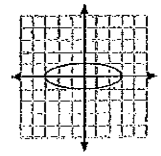
Write the equation of the circle graphed to the right.

a) $(x-1)^2+(y+2)^2=4$ b) $(x-1)^2+(y+2)^2=2$
c) $(x+1)^2+(y-2)^2=4$ d) $(x+1)^2+(y-2)^2=2$



Write the equation of the ellipse graphed to the right.

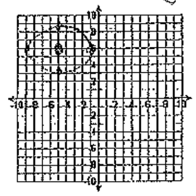
a) $\frac{x^2}{9}+\frac{y^2}{1}=1$ b) $\frac{x^2}{1}+\frac{y^2}{9}=1$
c) $\frac{x^2}{9}-\frac{y^2}{1}=1$ d) $\frac{x^2}{3}+\frac{y^2}{1}=1$



Graph the equation. Identify the vertices, co-vertices, and foci of the ellipse.

H 1. $\frac{(x+5)^2}{16}+\frac{(y-6)^2}{9}=1$

Center: $(-5,6)$
Vertices: $(-1,6)$; $(-9,6)$
Co-vertices: $(-5,3)$ $(-5,9)$
Foci:______;______ $(-5\pm\sqrt{7},6)$

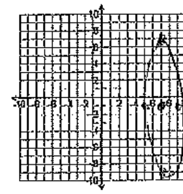


F: $(h\pm c,k)$

$c^2=a^2-b^2$
$c^2=16-9$
$c^2=7$
$c=\sqrt{7}$

V 2. $\frac{(x-7)^2}{4}+\frac{(y+1)^2}{64}=1$

Center: $(7,-1)$
Vertices: $(7,7)$; $(7,-9)$
Co-vertices: $(5,-1)$; $(9,-1)$
Foci:______;______ $(7,-1\pm2\sqrt{15})$



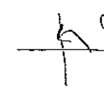
F: $(h,k\pm c)$

$c^2=64-4$
$c^2=60$
$c=2\sqrt{15}$

60
2 30
2 15
3 5

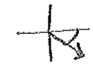
Examples: Sketch and state the quadrant.

A. ½ rotation counterclockwise



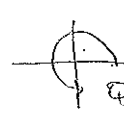
QA

B. $\frac{1}{6}$ rotation clockwise



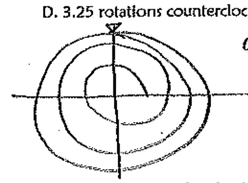
Q4

C. ¾ rotation counterclockwise



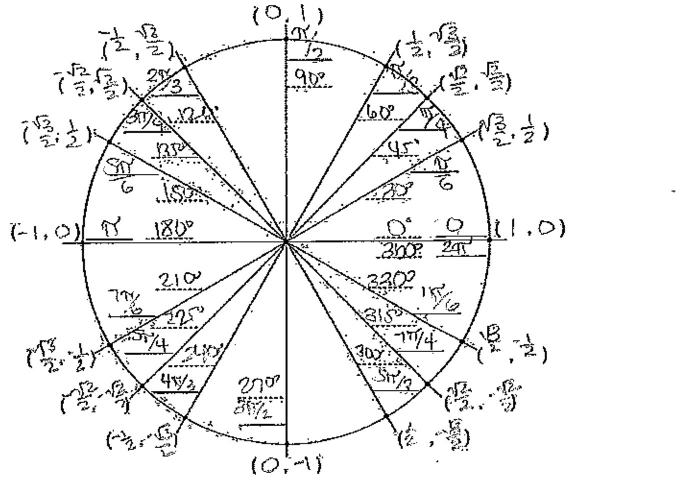
QA

D. 3.25 rotations counterclockwise



QA

- THE UNIT CIRCLE- Be able to fill one out fully- degrees, radians, ordered pairs



- Convert between degrees and DMS

AS to decimal

199°43'18" 199.72°

B. 59°49'3" 59.82°

C. 301°5'59" 301.10°

cimal to DMS

65.4921 65°29'31.56"

B. 177.0124 177°0'44.64"

C. 44.711 44°42'39.6"

- Convert between radians and degrees

grees to radians- State the quadrant it is in.

310° Q4 $\frac{31\pi}{18}$

B. 90° QA $\frac{\pi}{2}$

C. 250° Q3 $\frac{25\pi}{18}$

D. −85° Q4 $-\frac{17\pi}{36}$

dians to Degrees- state the quadrant it is in.

$\frac{19\pi}{10}$ 342° Q4

B. $\frac{8\pi}{11}$ 130.9° Q2

C. $\frac{17\pi}{8}$ 382.5° Q1

ecial Triangles

- Pythagorean Theorem
- 30-60-90 Triangles
- 45-45-90 Triangles
- SOH-CAH-TOA

g Functions

- Sin, cos, tan, csc, sec, tan
- Be able to find all the trig functions given one and/or given an ordered pair
- Quadrantal angles
- References angles

: Length

ea of a Sector

e and Cosine Graphs

Transformations $y = AsinP(\theta - S) + T$

- Amplitude, period, phase shift, vertical shift

Trig Identities

- Simplifying and verifying
- Sum and difference identities
- Double angle and half angle

Inverse Trig (arcsin or sin^{-1})

Solving Trig Equations (Basic, quadratic- formula or factor)

Solving Oblique Triangles

Polar Coordinates

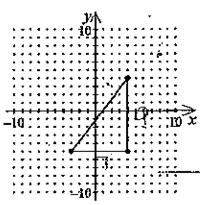
Sequences and Series

2/C Final Review Name: ___________

1. Find the distance between the points. $d=\sqrt{(5+6)^2+(-2-1)^2} = \sqrt{11^2+(-3)^2} = \sqrt{121+9} = \sqrt{130}$
$(-6, 1)$, $(5, -2)$

[A] $\sqrt{2} \approx 1.41$ [B] 130 [C] $\sqrt{130} \approx 11.40$ [D] 2

2. Find the length of the hypotenuse of the triangle.



$7^2+9^2=c^2$ $c^2=130$
$49+81=c^2$

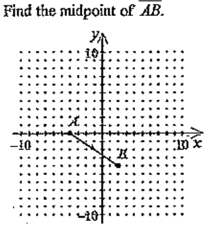
$\sqrt{130}$

[A] 1.41 [B] 9.06 [C] 7.07 [D] 11.4

3. Find the midpoint of the line segment connecting $(-9, -11)$ and $(12, -16)$. $\frac{-9+12}{2}, \frac{-11+-16}{2}$

[A] $(-3, 27)$ [B] $\left(\frac{3}{2}, -\frac{27}{2}\right)$ [C] $(3, -27)$ [D] $\left(-\frac{21}{2}, \frac{5}{2}\right)$ $\left(\frac{3}{2}, \frac{-27}{2}\right)$

4. Find the midpoint of $\overline{AB}$.



$(-4, 0)$ $(2, -4)$

$\frac{-4+2}{2}, \frac{0-4}{2} = -\frac{2}{2}, -\frac{4}{2}$

$(-1, -2)$

[A] $(-2, -1)$ [B] $(2, 1)$ [C] $(1, 2)$ [D] $(-1, -2)$

5. Find the slope of the line passing through the pair of points. $\frac{3+2}{2+7} = \frac{5}{9}$
$(-7, -2)$, $(2, 3)$

[A] $\frac{9}{5}$ [B] $\frac{7}{5}$ [C] $\frac{5}{9}$ [D] $-\frac{1}{5}$

6. Find the general form of the equation of the line that passes through the given point and has the indicated slope. $-3 = \frac{3}{5}(0)+b$ $b=-3$ $y=\frac{3}{5}x-3$
$(0, -3)$, $m=\frac{3}{5}$

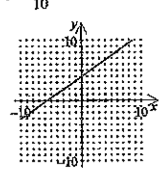
[A] $3x-5y-15=0$ [B] $5x+3y+9=0$ $\left(-\frac{3}{5}x+y=-3\right) \cdot 5$
[C] $-5x+3y+9=0$ [D] $3x+5y+15=0$

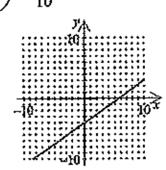
$3x-5y=15$

7. Find the slope-intercept form of the equation and its graph.
$-7x+10y=-40$

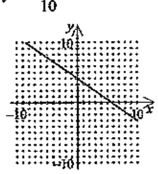
[A] $y=\frac{7}{10}x+4$ [B] $y=\frac{7}{10}x-4$ $10y=7x-40$

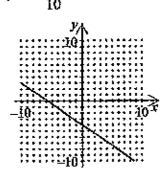
$y=\frac{7}{10}x-4$





[C] $y=-\frac{7}{10}x+4$ [D] $y=-\frac{7}{10}x-4$





8. Find the solution to the equation. [A] 13 [B] 11 [C] 7 [D] 5
$4=4(x-2)+1-3x$

$4=4x-8+1-3x$

$4=x-7$
$+7 \quad +7$

$9x^2(4x^2-1)=0$ $(2x-1)(2x+1)=0$

$9x^2=0$

$x=0$

9. Solve. $36x^4-9x^2=0$ [A] $\pm\frac{1}{4}$ [B] 0, ± 2 [C] 0, $\pm\frac{1}{2}$ [D] ± 4

10. Determine which equation represents y as a function of x.
[A] $y=1$ [B] $-6x^2+y^2=8$ [C] $x=8y^2$ [D] $x=1$

Evaluate the function at the specified value(s) of the independent variable and simplify.

11. $f(x)=|x|+5$; $f(-6)$ [A] -1 [B] 1 [C] -11 [D] 11

12. $f(x)=2x^2-\sqrt{-2x}$; $f(-5)$ [A] 54.472 [B] 40 [C] 45.838 [D] 57.071

13. $g(x)=\dfrac{x^2-3}{2x}$; $g(n-6)$ $\dfrac{(n-6)^2-3}{2(n-6)} = \dfrac{n^2-12n+36-3}{2n-12}$

[A] $\dfrac{n^2-12n+33}{2n-6}$ [B] $\dfrac{n^2-12n+33}{2n-12}$ [C] $\dfrac{n^2-3}{2n}-6$ [D] $\dfrac{n^2-9}{2n-12}$

Find the domain of the function.

14. $f(x)=\dfrac{x^2-4x-5}{x^2+10x+24}$ $(x+6)(x+4)$

[A] All real numbers $x\neq 5, -1$ [B] All real numbers $x\neq -4, -6$
[C] All real numbers $x\neq 4, 6$ [D] All real numbers $x\neq -5, 1$

15. $h(x)=\dfrac{3x}{x(x^2-64)}$

[A] All real numbers $x\neq \pm 8$ [B] All real numbers $x\neq \pm 64, 0$
[C] All real numbers $x\neq 8$ [D] All real numbers $x\neq \pm 8, 0$

16. $f(x)=\sqrt{-x-8}$ [A] $x\le -8$ [B] $x\ge -8$ [C] $x\le 0$ [D] $x\ge 0$

$-x-8\ge 0$
$+8 \quad +8$
$-x\ge 8$
$x\le -8$

17. Identify the equation of a quadratic function whose graph opens upward.

[A] $f(x)=-4(11x+4)^2$ [B] $f(x)=6(3x+9)$
[C] $f(x)=6x^2+4$ [D] $f(x)=-4x^2+3$

18. For the graph of the quadratic function, identify the direction of the opening and the coordinates of the vertex.
$f(x)=-(x+3)^2-3$

[A] Downward; (3, 3) [B] Upward; $(-3, -3)$
[C] Downward; $(-3, -3)$ [D] Upward; (3, 3)

19. Use synthetic division to divide.
$(-3z^5+3z^4+5z^3+z^2-3z-4)\div(z+2)$

$-2 \,|\, -3 \quad 3 \quad 5 \quad 1 \quad -3 \quad -4$
$\quad\quad 6 \quad -18 \quad 26 \quad -54 \quad 114$
$-3 \quad 9 \quad -13 \quad 27 \quad -57 \quad 110$

[A] $9z^4-13z^3+27z^2-57z+110$ [B] $-3z^5+9z^4-13z^3+27z^2-57z+110$
[C] $3z^4-9z^3+13z^2-27z+57+\dfrac{110}{z+2}$ [D] $-3z^4+9z^3-13z^2+27z-57+\dfrac{110}{z+2}$

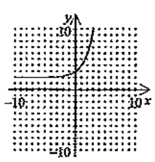
20. Which is the complex number in standard form?
$11i-8i^2$ $11i+8$
[A] $8-11i$ [B] $-8+11i$ [C] $19i$ [D] $8+11i$

21. Divide and write the result in standard form.
$\dfrac{4+3i}{9i}\cdot\dfrac{9i}{9i} = \dfrac{36i+27i^2}{81i^2} = \dfrac{36i-27}{-81} = \dfrac{-4i+3}{9}$

[A] $\dfrac{3-4i}{81}$ [B] $\dfrac{-27+4i}{9}$ [C] $\dfrac{3-4i}{9}$ [D] $\dfrac{-27+4i}{81}$

22. Evaluate the expression. $8^{\sqrt{3}}$ [A] 36.660 [B] 81.000 [C] 0.027 [D] 13.856

23. Identify the graph of the function.
$f(x)=2^x-2$

[A]

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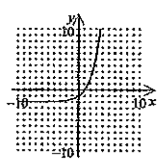
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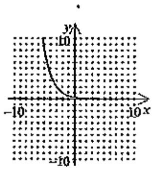
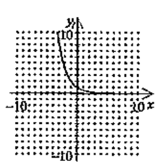

[D]


24. Use a calculator to evaluate the logarithm.
$\log_{10}1.07$
[A] 14.780 [B] 34.032 [C] 0.029 [D] 0.068

25. Identify the logarithmic equation written in exponential form.
$\log_{16}64=\frac{3}{2}$

[A] $64^{3/2}=16$ [B] $16^{3/2}=64$ [C] $\left(\frac{3}{2}\right)^{64}=16$ [D] $\left(\frac{3}{2}\right)^{16}=64$

Use the properties of logarithms to expand the expression. (Assume all variables are positive.)

26. $\log_b\sqrt[8]{\dfrac{x^2y^9}{z^7}}$ $\log_b \dfrac{x^{\frac{1}{4}}y^{\frac{9}{8}}}{z^{\frac{7}{8}}}$

[A] $\frac{1}{2}\log_b(x+y-z)$ [B] $\frac{1}{4}\log_b x+\frac{9}{8}\log_b y-\frac{7}{8}\log_b z$
[C] $\frac{1}{4}\log_b x+\frac{8}{9}\log_b y-\frac{8}{7}\log_b z$ [D] $\sqrt[8]{\dfrac{18(\log_b x)(\log_b y)}{7\log_b z}}$

27. $\log_5\dfrac{x^9}{\sqrt{y}}$

[A] $9\log_5 x-\frac{1}{2}\log_5 y$ [B] $9\log_5 x+\frac{1}{2}\log_5 y$
[C] $\log_5 x^9+2\log_5 y$ [D] $\log_5 x^9-2\log_5 y$

28. Condense the expression to the logarithm of a single quantity.
$6\log_{10}x+5\log_{10}(x-4)$
[A] $\log_{10}x^6(x-4)^5$ [B] $\log_{10}x(x-4)$ [C] $\log_{10}x(x-4)^{30}$ [D] None of these

29. Solve for x. $\left(\frac{1}{4}\right)^x=64$ [A] $\frac{1}{3}$ [B] $-\frac{1}{3}$ [C] 3 [D] -3

$\dfrac{x\log\frac{1}{4}}{\log\frac{1}{4}}=\dfrac{\log 64}{\log\frac{1}{4}}$

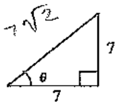
30. Use the period of the function to select the expression that has the same value as the given expression.
$\sin\left(-\dfrac{23\pi}{4}\right)$ $45°$ ref ∠

[A] $\sin\frac{\pi}{2}$ [B] $\sin\frac{\pi}{8}$ [C] $\sin\frac{\pi}{4}$ [D] $\sin\frac{3\pi}{4}$

31. Use a calculator to evaluate the expression.
$\sin(-3.8)$
[A] 1.6344 [B] -0.0663 [C] -15.0889 [D] 0.6119

Find the exact value of the sine and cosine functions of the angle θ given in the figure. (Use the Pythagorean Theorem to find the third side of the triangle.)

32.



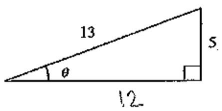
$7^2+7^2=c^2$
$49+49=c^2$
$c^2=98$

$c=\sqrt{98} = 7\sqrt{2}$
$(2)\,49$

$\sin\theta = \frac{7}{7\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{7\sqrt{2}}{14} = \frac{\sqrt{2}}{2}$

[A] $\sin\theta=\sqrt{2}$, $\cos\theta=\sqrt{2}$
[B] $\sin\theta=\frac{\sqrt{2}}{2}$, $\cos\theta=\frac{\sqrt{2}}{2}$
[C] $\sin\theta=1$, $\cos\theta=1$
[D] $\sin\theta=\frac{\sqrt{5}}{2}$, $\cos\theta=\frac{\sqrt{5}}{2}$

33.



[A] $\sin\theta=\frac{5}{12}$, $\cos\theta=\frac{12}{5}$
[B] $\sin\theta=\frac{5}{13}$, $\cos\theta=\frac{12}{13}$
[C] $\sin\theta=\frac{12}{13}$, $\cos\theta=\frac{5}{13}$
[D] $\sin\theta=\frac{13}{12}$, $\cos\theta=\frac{13}{5}$

Find the amplitude and the period of the function.

34. $y=-2.5\sin 2x$

[A] Amplitude $=2.5$, Period $=2$
[B] Amplitude $=-2.5$, Period $=\pi$
[C] Amplitude $=-2.5$, Period $=2$
[D] Amplitude $=2.5$, Period $=\pi$

Per $= \frac{2\pi}{2} = \pi$

Find the amplitude and the period of the function.

35. $y=-4\cos\frac{2x}{3}$

[A] Amplitude $=4$, Period $=3\pi$
[B] Amplitude $=-4$, Period $=\frac{2}{3}$
[C] Amplitude $=-4$, Period $=3\pi$
[D] Amplitude $=4$, Period $=\frac{2}{3}$

Per $= \frac{2\pi}{\frac{2}{3}}$

$= \frac{2\pi}{1} \cdot \frac{3}{2} = \frac{6\pi}{2} = 3\pi$

36. Give the following polynomial: $f(x)=x^4-7x^2-6x$

a. List all the possible rational zeros.

no "p"

$x(x^3-7x-6)=0$

$x=0$

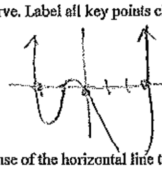
Leading coefficient is 1, try −1

b. Find all the roots.

$x=-2,-1,0,3$

$$\begin{array}{r|rrrr} -1 & 1 & 0 & -7 & -6 \\ & & -1 & 1 & 6 \\ \hline & 1 & -1 & -6 & 0 \end{array}$$

c. Sketch the curve. Label all key points clearly.



$x^2-x-6=0$

$(x-3)(x+2)=0$

$x=3 \quad x=-2$

37. Explain the use of the horizontal line test.

Used to determine if a function is one-to-one

38. Explain the use of the vertical line test.

used to determine if a graph is a function

39. How do you determine whether a function is odd or even?

degree - even or odd

40. Find the inverse of the following functions.

a. $f(x)=\frac{3}{x}$

$y=\frac{3}{x}$

$y\ x=\frac{3}{y}\cdot y$

$xy=3$

$\frac{xy}{x}=\frac{3}{x} \quad y=\frac{3}{x}$

b. $f(x)=3x-5$

$y=3x-5$

$x=3y-5$

$+5 \quad +5$

$\frac{x+5}{3}=\frac{3y}{3}$

$y=\frac{x+5}{3}$

41. Given $f(x)=3x-5$ *and* $g(x)=x^2+7$, evaluate the following:

a. $f\circ g(x)$

$3(x^2+7)-5$

$3x^2+21-5$

$3x^2+16$

b. $g\circ f(x)$

$(3x-5)^2+7$

$9x^2-30x+25+7$

$9x^2-30x+32$

c. $g\circ g(4)$

4^2+7

$=16+7$

$=23$

23^2+7

536

42. Write the rule for the following sequences:

a. arithmetic: $a_5=12$; $a_8=24$

$t_n=t_1+(n-1)d$

b. geometric: $a_3=1$; $a_5=16$

$t_n=t_1\cdot r^{n-1}$

43. Given the rectangular coordinates of a point, plot the point and find two sets of polar coordinates for the point, $0\le\theta\le2\pi$

a. (-1, 1)

$\tan\theta=\frac{1}{-1}$

$r=\pm\sqrt{x^2+y^2}$

$=\pm\sqrt{1+1}$

$=\pm\sqrt{2}$

$=-1$

$\tan\theta=-1$

$\theta=$

$(2,-45°)(-2,135°)$

b. (3, -1)

$r=\pm\sqrt{9+1}$

$=\pm\sqrt{10}$

$\tan\theta=-\frac{1}{3}$

$(\sqrt{10},18.4°)(-\sqrt{10},161.57°)$

c. (5, 12)

$r=\pm\sqrt{25+144}=\sqrt{169}=\pm13$

$\tan\theta=\frac{12}{5}$

$(13,67.3°)(-13,247.3°)$

44. Convert the following polar coordinates to rectangular coordinates:

a. $\left(-1,-\frac{3\pi}{4}\right)$

$x=r\cos\theta$

$=-1\cos-135$

$y=r\sin\theta$

$=-1\sin-135$

$(0.71, 0.71)$

b. $\left(4,\frac{3\pi}{2}\right)$

$x=4\cos 270$

$y=4\sin 270$

$(0,-4)$

c. $\left(0,-\frac{3\pi}{6}\right)$

$x=0\cos\theta$

$(0,0)$

45. Convert the rectangular equation to polar form:

a. $x^2+y^2=9$

$r^2=9$

$r=\pm3$

b. $y=4$

$r\sin\theta=4$

$r=\frac{4}{\sin\theta}$

$r=4\csc\theta$

c. $x=10$

$r\cos\theta=10$

$r=\frac{10}{\cos\theta}$

$r=10\sec\theta$

46. Convert the polar equation to rectangular form:

a. $r=4\sin\theta$

$\sqrt{x^2+y^2}=4\left(\frac{y}{\sqrt{x^2+y^2}}\right)\cdot\sqrt{x^2+y^2}$

$x^2+y^2=4y$

$x^2+y^2-4y+4=4$

$x^2+(y-2)^2=4$

b. $r=4$ cos (theta)

$\sqrt{x^2+y^2}=4\left(\frac{x}{\sqrt{x^2+y^2}}\right)$

$x^2+y^2=4x$

$x^2-4x+4+y^2=4$

$(x-2)^2+y^2=4$

c. $r=2\csc\theta$

$y\cdot\sqrt{x^2+y^2}=2\left(\frac{\sqrt{x^2+y^2}}{y}\right)y$

$(y\cdot\sqrt{x^2+y^2})^2=(2\sqrt{x^2+y^2})^2$

$\frac{y^2(x^2+y^2)}{x^2+y^2}=\frac{4(x^2+y^2)}{x^2+y^2}$

$y^2=4$

$y=\pm2$

47. Simplify using the properties of logarithms:

a. $\ln e^{2x+1}$

$2x+1$

b. $\log_5 25^{2x}=x$

$5^x=25^{2x}$

$5^x=5^{2(2x)}$

$x=4x$

$-x \quad -x$

$0=3x$

$x=0$

c. $5+\ln e^{3x^2+5x+6}$

$5+3x^2+5x+6$

$3x^2+5x+11$

48. Solve the following logarithmic equations:

a. $\ln x+\ln(x-2)=\ln e^3$

$x(x-2)=3$

$x^2-2x=3$

$x^2-2x-3=0$

$(x-3)(x+1)=0$

$x=3 \quad x=-1$

b. $\log_5 x+\log_5(x+2)=\log_5(x+6)$

$x(x+2)=x+6$

$x^2+2x=x+6$

$-x-6 \quad -x \quad -6$

$x^2+x-6=0$

$(x+3)(x-2)=0$

reject

$x=-3 \quad \boxed{x=2}$

c. $\ln 4x=1$

$\frac{e^1}{4}=\frac{4x}{4}$

$x=0.68$

d. $5^{x+2}=3^{2x-1}$

$\frac{x+2(\log 5)}{\log 3}=\frac{2x-1(\log 3)}{\log 3}$

$(x+2)(1.46)=2x-1$

$1.46x+2.93=2x-1$

$-1.46x \quad +1 \quad -1.46x \quad +1$

$\frac{3.93}{0.54}=\frac{0.54x}{0.54}$

$x=7.28$

49. A deposit of $160,000 is compounded monthly at an annual percentage rate of 6.75%. How long will it take for the money to triple?

$\frac{480000}{160000}=\frac{160000}{160000}\left(1+\frac{0.0675}{12}\right)^{12t}$

$3=1.005625^{12t}$

$\frac{\log 3}{\log 1.005625}=\frac{12t\log 1.005625}{\log 1.005625}$

$\frac{12t}{12}=\frac{195.86}{12}$

$t=16.3$ yrs

50. A deposit of $5,500 is compounded continuously at an annual percentage rate of 7%. How long will it take until you have $160,000?

$\frac{160,000}{5500}=\frac{5500e^{0.07t}}{5500}$

$\frac{\ln 29.1}{0.07}=\frac{0.07t\ln e}{0.07}$

$t=48.15$ yrs

51. An investment of $10,000 is compounded continuously. What annual percentage rate will produce a balance of $25,000 in 10 years?

$\frac{25000}{10000}=\frac{10000e^{r\cdot10}}{10000}$

$2.5=e^{10r}$

$\frac{\ln 2.5}{10}=\frac{10r\ln e}{10}$

$r=0.0916$

$=9.16\%$

52. Evaluate $f(x)=3x^2+5x-6$ using the difference quotient.

$\frac{f(x+h)-f(x)}{h} \quad \frac{3(x+h)^2+5(x+h)-6-(3x^2+5x-6)}{h}$

$\frac{3x^2+6xh+3h^2+5x+5h-6-(3x^2+5x-6)}{h}$

$=\frac{6xh+3h^2+5h}{h}=6x+3h+5$

1. Evaluate the following using your calculator if necessary. Remember the quadrants for which the inverse functions are defined.

a. $\sin 5.36$ — 0.0934
b. $\sec 75°$ — 3.86
c. $\tan\frac{9\pi}{2}$ — $\varnothing$

d. $\sin^{-1}(1.5686)$ — $\varnothing$
e. $\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)$ — 150°
f. $\arctan(9.85)$ — 84.2°, 275.8°

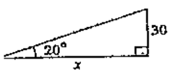
2. Find the following without using your calculator:

a. $\sin\left(-\frac{8\pi}{3}\right)$ — $-480°$, $-120°$, $-\frac{\sqrt{3}}{2}$
b. $\cos\frac{3\pi}{4}$ — $-\frac{\sqrt{2}}{2}$
c. $\tan\frac{11\pi}{6}$ — $-\frac{\sqrt{3}}{3}$

d. $\csc\left(\arccos\frac{x}{\sqrt{2}}\right)$
e. $\sin(\arctan x)$
f. $\tan\left(\arcsin\frac{x}{2}\right)$

3. Solve for the remaining parts of the following triangles

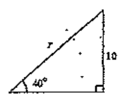
a.



$\tan 20 = \frac{30}{x}$

$x = 82.4$

b.

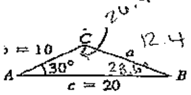


$\frac{\sin 40}{1} = \frac{10}{r}$

$r\sin 40 = 10$

$r = 15.6$

c.



$a^2 = 10^2 + 20^2 - 2(10)(20)\cos 30$

$a^2 = 100 + 400 - 400\cos 30$

$a^2 = 500 - 400\cos 30$

$a^2 = 153.6$

$a = 12.4$

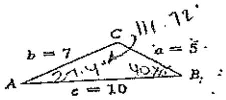
$\frac{\sin 30}{12.4} = \frac{\sin B}{10}$

$12.4\sin B = 10\sin 30$

$\sin B = 0.40$

$B = 23.6°$

d.



$10^2 = 7^2 + 5^2 - 2(7)(5)\cos C$

$100 = 49 + 25 - 70\cos C$

$100 = 74 - 70\cos C$

$26 = -70\cos C$

$\cos C = -0.37$

$C = 111.72°$

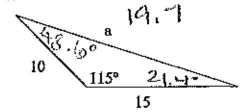
$\frac{\sin 111.72}{10} = \frac{\sin A}{5}$

$10\sin A = 5\sin 111.72$

$\sin A = 0.46$

$A = 27.4°$

e.



$a^2 = 10^2 + 15^2 - 10(15)\cos 115$

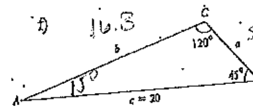
$a^2 = 100 + 225 - 150\cos 115$

$a^2 = 388.39$

$a = 19.7$

$\frac{\sin 115}{19.7} = \frac{\sin B}{15}$ $B = 43.6°$

f.



$\frac{\sin 120}{20} = \frac{\sin 45}{b}$

$b = 16.3$

$\frac{\sin 120}{20} = \frac{\sin 15}{a}$

$a = 5.98$

4. Verify the following identities.

a. $\frac{\sec^2\theta - 1}{\sec^2\theta} = \sin^2\theta$

$\frac{\tan^2\theta}{\sec^2\theta}$

$= \frac{\sin^2\theta}{\cos^2\theta} \cdot \frac{\cos^2\theta}{1}$

$\sin^2\theta = \sin^2\theta$

b. $\sec y + \tan y = \frac{\cos y}{1-\sin y}$

$\frac{1}{\cos y} + \frac{\sin y}{\cos y}$

$\frac{1+\sin y}{\cos y}$

c. $\frac{\cot^2\theta}{1+\csc\theta} = \frac{1-\sin\theta}{\sin\theta}$

$1 + \frac{1}{\sin\theta}$

$\frac{\sin\theta}{\sin\theta} + \frac{1}{\sin\theta}$

$\frac{\sin\theta + 1}{\sin\theta}$

$\frac{\cos^2\theta}{\sin^2\theta} \div \frac{\sin\theta+1}{\sin\theta}$

$\frac{\cos^2\theta}{\sin^2\theta} \cdot \frac{\cancel{\sin\theta}}{\sin\theta+1}$

$\frac{\cos^2\theta}{\sin\theta(\sin\theta+1)} = \frac{1-\sin^2\theta}{(1+\sin\theta)\sin\theta}$

$= \frac{(1-\sin\theta)(1+\sin\theta)}{\sin\theta\cancel{(1+\sin\theta)}} \Rightarrow \frac{1-\sin\theta}{\sin\theta}$

d. $\csc x - \sin x = \cos x \cot x$

$\frac{1}{\sin x} - \frac{\sin x}{1} = \frac{\cos x}{1} \cdot \frac{\cos x}{\sin x}$

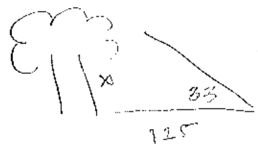
$\frac{1}{\sin x} - \frac{\sin^2 x}{\sin x} = \frac{\cos^2 x}{\sin x}$

$\frac{\cos^2 x}{\sin x} = \frac{\cos^2 x}{\sin x}$

cos sec cot

sin csc tan

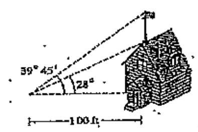
56. The length of the shadow of a tree is 125 feet when the angle of elevation of the sun is 33°. Approximate the height of the tree.



$\frac{\tan 33}{1} = \frac{x}{125}$

$x = 81.18$ ft

57. From a point 100 feet in front of a public library, the angles of elevation to the base of the flagpole and to the top of the pole are 28° and 39° 45', respectively. The flagpole is mounted on the front of the library's roof. Find the height of the pole.



$x = 83.17$

$39°45' = 39.75°$

$\tan 39.75 = \frac{x}{100}$

$\tan 28 = \frac{y}{100}$ $y = 53.17$

$83.17 - 53.17$

$\boxed{30 \text{ ft}}$